答案详解(第37~40期)

主编 王 静 终审 李双孝
责编 陈记梅 照排 仇晓凤

同步分层能力测试题(四)

基础练

一、选择题

1~6.DBAAAD.

提示:

1.可得$y=\sin[2(x-\frac{\pi}{4})+\frac{\pi}{4}]=\sin(2x-\frac{\pi}{4})$为非奇非偶函数.

故选D.

2.函数$y=\sin(2x-\frac{\pi}{3})$的图象向左平移$\frac{\pi}{6}$个单位长度,可得函数$y=\sin[2(x+\frac{\pi}{6})-\frac{\pi}{3}]$的图象,整理得$y=\sin2x$.故选B.

3.经分析知,$\frac{\pi}{3}$是函数$f(x)$周期的整数倍,即$nT=\frac{\pi}{3}$($n\in\mathbf{Z},n\neq0$).又函数$f(x)$的周期$T=\frac{2\pi}{\omega}$,所以$\omega=6n$,当$n=1$时,正数ω的最小值为6.故选A.

4.由题得图象变换最后得到的解析式为$y=\sin2(x-\frac{\pi}{6})=\sin(2x-\frac{\pi}{3})$,令$2x-\frac{\pi}{3}=k\pi+\frac{\pi}{2}$,$k\in\mathbf{Z}$,$\therefore x=\frac{k\pi}{2}+\frac{5}{12}\pi$,令$k=-1$,所以$x=-\frac{\pi}{12}$.故选A.

5.由题可知,图象的周期为π,于是由$T=\frac{2\pi}{\omega}$,得$\omega=2$.由于$y=2\sin(2x+\theta)$为偶函数,有$\theta=k\pi+\frac{\pi}{2}$,$k\in\mathbf{Z}$,$2\sin\theta=2$,即$\omega=2,\theta=\frac{\pi}{2}$.

故选A.

6.根据函数的图象知,振幅$A=\frac{2}{3}$,周期$T=\frac{5\pi}{12}-(-\frac{7\pi}{12})=\pi$,即$\frac{2\pi}{\omega}=\pi$,解得$\omega=2$.所以$x=-\frac{\pi}{12}$时,$\omega x+\varphi=2\times(-\frac{\pi}{12})+\varphi=\frac{\pi}{2}+2k\pi$,$k\in\mathbf{Z}$,解得$\varphi=\frac{2\pi}{3}+2k\pi$,$k\in\mathbf{Z}$,所以函数$y$的一个解析式为$y=\frac{2}{3}\sin(2x+\frac{2\pi}{3})$.故选D.

二、填空题

7.$\frac{1}{2}$;

8.4π.

提示:

7.将函数$y=\sin2x$的图象向左平移$\varphi(0<\varphi<\frac{\pi}{2})$个单位,得到函数$y=\sin[2(x+\varphi)]$的图象,依题意知$2\varphi=1$,$\therefore\varphi=\frac{1}{2}$.

8.最小值为$\frac{T}{2}=4\pi$.

三、解答题

9.解:(1)列表:

$\frac{x}{2}+\frac{\pi}{6}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
x	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{3}$	$\frac{8\pi}{3}$	$\frac{11\pi}{3}$
y	3	6	3	0	3

图象如图1所示:

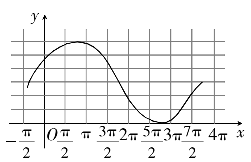

图1

(2)周期为4π;函数$f(x)$的单调减区间为$[\frac{2\pi}{3}+4k\pi,\frac{8\pi}{3}+4k\pi]$,$k\in\mathbf{Z}$.

(3)函数$f(x)=3\sin(\frac{x}{2}+\frac{\pi}{6})+3$的图象由函数$y=\sin x$在$[0,2\pi]$上的图象先向左平移$\frac{\pi}{6}$个单位;然后纵坐标不变,横坐标伸长为原来的2倍;然后横坐标不变,纵坐标伸长为原来的3倍;最后沿y轴向上平移3个单位得到.

10.解:(1)由$f(x)$的最小值为-2,且$A>0$,所以$A=2$.

因为$f(\frac{1}{3})=-2$,所以$\cos(\frac{\pi}{3}+\varphi)=-1$.

由$0<\varphi<\pi$,可得$\frac{\pi}{3}<\frac{\pi}{3}+\varphi<\frac{4\pi}{3}$,

所以$\frac{\pi}{3}+\varphi=\pi$,所以$\varphi=\frac{2\pi}{3}$.

故$f(x)$的解析式为$f(x)=2\cos(\pi x+\frac{2\pi}{3})$.

(2)$f(x)=2\cos(\pi x+\frac{2\pi}{3})$,

由$-\pi+2k\pi\leqslant\pi x+\frac{2\pi}{3}\leqslant2k\pi$,$k\in\mathbf{Z}$,

解得$-\frac{5}{3}+2k\leqslant x\leqslant-\frac{2}{3}+2k$,$k\in\mathbf{Z}$.

$\therefore$函数$f(x)$的单调递增区间为$[-\frac{5}{3}+2k,-\frac{2}{3}+2k]$,$k\in\mathbf{Z}$.

11.解:(1)由已知,得当$x=\frac{\pi}{6}$时,$2\omega x+\frac{\pi}{3}=2\omega\cdot\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{2}$,得$\omega=\frac{1}{2}$.

(2)由(1),得$f(x)=\sin(x+\frac{\pi}{3})+\frac{\sqrt{3}}{2}+a$.

当$x\in[-\frac{\pi}{3},\frac{5\pi}{6}]$时,$x+\frac{\pi}{3}\in[0,\frac{7\pi}{6}]$,

得$-\frac{1}{2}\leqslant\sin(x+\frac{\pi}{3})\leqslant1$.

$\therefore$当$x+\frac{\pi}{3}=\frac{7\pi}{6}$时,$\sin(x+\frac{\pi}{3})$取得最小值$-\frac{1}{2}$,

则$\sqrt{3}=-\frac{1}{2}+\frac{\sqrt{3}}{2}+a$,得$a=\frac{\sqrt{3}+1}{2}$.

12.解:结合题意和图象,可知$\begin{cases}\dfrac{\frac{2\pi}{\omega}}{4}=6,\\ A\sin6\omega=4\sqrt{3},\end{cases}$ 解此方程组,得$\begin{cases}\omega=\frac{\pi}{12},\\ A=4\sqrt{3},\end{cases}$ 于是$y=4\sqrt{3}\sin\frac{\pi}{12}x(x\in[0,8])$.

当$x=8$时,$y=4\sqrt{3}\sin\frac{8\pi}{12}=6$,$\therefore$点$M$的坐标为$(8,6)$.

所以$MP=\sqrt{(8-16)^2+(6-0)^2}=10(\text{km})$.

提升练

一、选择题

1~6.AAACAD.

提示:

1.为了得到函数的图象$y=\cos\frac{x}{5}$,只需把余弦曲线上的所有点横坐标伸长到原来的5倍,纵坐标不变.故选A.

2.可得$\frac{T}{4}=\frac{\pi}{8}$,则$T=\frac{\pi}{2}$.

故选A.

3.把函数$y=\cos(2x+\frac{\pi}{6})=\sin(2x+\frac{\pi}{6}+\frac{\pi}{2})=\sin(2x+\frac{2}{3}\pi)$的图象向右平移$\frac{\pi}{3}$个单位,得到$y=\sin[2(x-\frac{\pi}{3})+\frac{2}{3}\pi]=\sin2x$的图象.

故选A.

4.由题意,可知$A=\frac{f(x)_{\min}-f(x)_{\max}}{2}=\frac{-\frac{1}{2}-\frac{1}{2}}{2}=-\frac{1}{2}$,$B$,$C$两点横坐标之差的绝对值为周期$T$的一半,则$T=2\times(\frac{5}{2}-\frac{3}{2})=2$,$\therefore\omega=\frac{2\pi}{T}=\pi$,因此$A=-\frac{1}{2}$,$\omega=\pi$.故选C.

5.函数$f(x)=\sin(2x+\varphi)(0<\varphi<\pi)$的图象过$(\frac{7\pi}{12},-1)$,即$-1=\sin(\frac{7\pi}{6}+\varphi)\Rightarrow\varphi=\frac{\pi}{3}$,即把$f(x)=\sin(2x+\frac{\pi}{3})$的图象向右平移$\frac{\pi}{6}$个单位得到$g(x)=\sin2x$的图象.故选A.

6.将函数$y=\sin(x+\frac{\pi}{6})$的图象上各点的横坐标变为原来的$\frac{1}{2}$,可得$y=\sin(2x+\frac{\pi}{6})$的图象,$\because-\frac{\pi}{4}\leqslant x\leqslant\frac{\pi}{2}$,$\therefore-\frac{\pi}{3}\leqslant2x+\frac{\pi}{6}\leqslant\frac{7\pi}{6}$,$\therefore y=\sin(2x+\frac{\pi}{6})$的最大值为1,最小值为$-\frac{\sqrt{3}}{2}$.故选D.

二、填空题

7.$\frac{5\pi}{6}$;　8.$\frac{14}{3}$.

提示:

7.将函数$y=\cos(2x+\varphi)$的图象向右平移$\frac{\pi}{2}$个单位后得到$y=\cos[2(x-\frac{\pi}{2})+\varphi]=\cos(2x-\pi+\varphi)=\sin(\frac{\pi}{2}+2x-\pi+\varphi)=\sin(2x+\varphi-\frac{\pi}{2})$.

$\because-\pi<\varphi\leqslant\pi$,$\varphi-\frac{\pi}{2}=\frac{\pi}{3}$,$\therefore\varphi=\frac{5\pi}{6}$.

8.可得$x=\frac{1}{2}(\frac{\pi}{6}+\frac{\pi}{3})=\frac{\pi}{4}$为第四点的横坐标,则$\frac{\pi}{4}\omega+\frac{\pi}{3}=\frac{3\pi}{2}$,得$\omega=\frac{14}{3}$.

三、解答题

9.解:(1)由图象,可知$A=2$,周期$T=\frac{4}{3}[\frac{5\pi}{12}-(-\frac{\pi}{3})]=\pi$,$\therefore\frac{2\pi}{\omega}=\pi$,$\omega>0$,则$\omega=2$,从而$f(x)=2\sin(2x+\varphi)$.代入点$(\frac{5\pi}{12},2)$,得$\sin(\frac{5\pi}{6}+\varphi)=1$,则$\frac{5\pi}{6}+\varphi=\frac{\pi}{2}+2k\pi$,$k\in\mathbf{Z}$,即$\varphi=-\frac{\pi}{3}+2k\pi$,$k\in\mathbf{Z}$.又$|\varphi|<\frac{\pi}{2}$,则$\varphi=-\frac{\pi}{3}$,$\therefore f(x)=2\sin(2x-\frac{\pi}{3})$.

(2)$\because x\in[0,\frac{\pi}{2}]$,则$2x-\frac{\pi}{3}\in[-\frac{\pi}{3},\frac{2\pi}{3}]$,

$\therefore$当$2x-\frac{\pi}{3}=\frac{\pi}{2}$,即$x=\frac{5\pi}{12}$时,$f(x)_{\max}=2$;

当$2x-\frac{\pi}{3}=-\frac{\pi}{3}$,即$x=0$,$f(x)_{\min}=-\sqrt{3}$.

10.解:(1)由表格中的数据,得$A=2$,$B=1$,$T=\frac{11\pi}{6}-(-\frac{\pi}{6})=2\pi$,$\omega=\frac{2\pi}{T}=1$.

当$x=\frac{5\pi}{6}$时,$\omega x+\varphi=\frac{5\pi}{6}+\varphi=\frac{\pi}{2}$,得$\varphi=-\frac{\pi}{3}$.

故$f(x)=2\sin(x-\frac{\pi}{3})+1$.

(2)$g(x)=2\sin(3x-\frac{\pi}{3})$,令$t=3x-\frac{\pi}{3}$,由$x\in[0,\frac{\pi}{3}]$,得$t\in[-\frac{\pi}{3},\frac{2\pi}{3}]$.

要使$2\sin t=\frac{m}{2}$,即$\sin t=\frac{m}{4}$在$[-\frac{\pi}{3},\frac{2\pi}{3}]$上恰有两个不同的解,作图象如图2所示,则有$\frac{\sqrt{3}}{2}\leqslant\frac{m}{4}<1$.

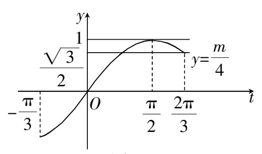

图2

故m的取值范围为$[2\sqrt{3},4)$.

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37期《堂堂清》

1.1.1 任意角

1.C;　2.C;　3.B;　4.D;　5.A;　6.B;　7.C;　8.C.

1.1.2 弧度制

1.$-\frac{11\pi}{6}$;　2.$\frac{3}{2}$;　3.B;　4.B;

5.$\frac{\pi}{6}$;　6.B;　7.D.

1.2.1 任意角的三角函数

1.C;　2.A;　3.D;　4.B;　5.B;　6.$-\frac{5}{13}$.

1.2.2 同角三角函数的基本关系

1.B;　2.D;　3.B;

4.$\frac{2}{5}$;　5.0.

6.解:(1)由$\tan\theta+\frac{1}{\tan\theta}=3$,得$\sin\theta\cos\theta=\frac{1}{3}$.

$\because\pi<\theta<\frac{5}{4}\pi$,$\therefore\cos\theta<\sin\theta<0$.

$\because(\sin\theta+\cos\theta)^2=\frac{5}{3}$,$\therefore\sin\theta+\cos\theta=-\frac{\sqrt{15}}{3}$.

(2)又$(\sin\theta-\cos\theta)^2=\frac{1}{3}$,$\cos\theta<\sin\theta<0$,

$\therefore\sin\theta-\cos\theta=\frac{\sqrt{3}}{3}$.

同步分层能力测试题(一)

基础练

一、选择题

1~6.DACAAB.

提示:

1.设扇形的半径是r,由扇形的弧长公式$l=|\alpha|\cdot r$,得$\frac{\pi}{2}=\frac{\pi}{4}\cdot r$,解得$r=2$.故选D.

2.钝角的范围为$90°<\theta<180°$,即钝角是第二象限角,故A正确;$-200°$是第二象限角,$60°$是第一象限角,$-200°<60°$,故B错误;由钝角的范围可知C错误;$-180°<-165°<-90°$,$-165°$是第三象限角,D错误.故选A.

3.区域为$\frac{\pi}{4}\leqslant\alpha\leqslant\frac{\pi}{2}$及$\pi+\frac{\pi}{4}\leqslant\alpha\leqslant\pi+\frac{\pi}{2}$.

故选C.

4.因为$585°-225°=360°$,所以$585°$与$225°$终边相同.故选A.

5.可得$\begin{cases}3a-9\leqslant0,\\ a+2>0,\end{cases}$解得$-2<a\leqslant3$.

故选A.

6.$\because$点$P(-\sin\theta,\cos\theta)$位于第三象限,$\therefore-\sin\theta<0$,$\cos\theta<0$,$\therefore\sin\theta>0$,$\cos\theta<0$,$\therefore\theta$是第二象限的角.

故选B.

二、填空题

7.-10;　8.$-\frac{5}{11}$.

提示:

7.由点$P(-1,2)$在角α的终边上,得$\sin\alpha=\frac{2\sqrt{5}}{5}$,$\cos\alpha=-\frac{\sqrt{5}}{5}$,$\tan\alpha=-2$,

所以$\frac{\tan\alpha}{\cos^2\alpha}=-10$.

8.原式分子分母同时除以$\cos\alpha$得到$\frac{1+2\tan\alpha}{2-3\tan\alpha}$,将$\tan\alpha=-3$代入上式,求得值为$-\frac{5}{11}$.

三、解答题

9.解:$\alpha=60°=\frac{\pi}{3}\text{rad}$,半径$R=6$,

(1)$\overset{\frown}{AB}$的长$l=\frac{\pi}{3}\times6=2\pi$.

(2)$S_{扇形OAB}=\frac{1}{2}lR=\frac{1}{2}\times2\pi\times6=6\pi$,因为$\triangle OAB$为正三角形,所以$S_{\triangle OAB}=\frac{\sqrt{3}}{4}R^2=\frac{\sqrt{3}}{4}\times6^2=9\sqrt{3}$.

$\therefore$弓形OAB的面积$S_{弓形OAB}=S_{扇形OAB}-S_{\triangle OAB}=6\pi-9\sqrt{3}$.

10.解:(1)$f(x)=-(x-1)^2+1+a(0\leqslant x\leqslant3)$,故$m=f(1)=1+a$,$n=f(3)=-3+a$.

(2)由(1)知角θ的终边经过点$P(a,a)$.

当$a>0$时,$r=\sqrt{a^2+a^2}=\sqrt{2}a$,得$\sin\theta=\frac{a}{\sqrt{2}a}=\frac{\sqrt{2}}{2}$,$\cos\theta=\frac{a}{\sqrt{2}a}=\frac{\sqrt{2}}{2}$,$\tan\theta=\frac{a}{a}=1$,则$\sin\theta+\cos\theta+\tan\theta=1+\sqrt{2}$;

当$a<0$时,$r=\sqrt{a^2+a^2}=-\sqrt{2}a$,得$\sin\theta=\frac{a}{-\sqrt{2}a}=-\frac{\sqrt{2}}{2}$,$\cos\theta=\frac{a}{-\sqrt{2}a}=-\frac{\sqrt{2}}{2}$,$\tan\theta=\frac{a}{a}=1$,则$\sin\theta+\cos\theta+\tan\theta=1-\sqrt{2}$.

11.解:(1)依题设,得$\begin{cases}\pi<2\theta<\frac{3\pi}{2},\\ 14\theta=2k\pi(k\in\mathbf{Z}),\end{cases}$ 则$\pi<\frac{2k\pi}{7}<\frac{3\pi}{2}$,即$\frac{7}{2}<k<\frac{21}{4}$,得$k=4$或5.

故$\theta=\frac{4\pi}{7}$,或$\theta=\frac{5\pi}{7}$.

(2)当$\theta=\frac{4\pi}{7}$时,弧长$l=\frac{4\pi}{7}\times1=\frac{4\pi}{7}$,扇形面积$S=\frac{2\pi}{7}$;

当$\theta=\frac{5\pi}{7}$时,弧长$l=\frac{5\pi}{7}$,扇形面积$S=\frac{5\pi}{14}$.

12.解:(1)证明:由已知得$1+\sin\alpha=\cos\beta(1-\cos\alpha)$,$1-\sin\alpha=\sin\beta(1-\cos\alpha)$.

将上述两个等式两边分别平方后相加,得$(1+\sin\alpha)^2+(1-\sin\alpha)^2=(\cos^2\beta+\sin^2\beta)(1-\cos\alpha)^2=(1-\cos\alpha)^2$.

$\therefore$等式成立.

(2)由(1),得$2+2\sin^2\alpha=1-2\cos\alpha+\cos^2\alpha$,即$2+2(1-\cos^2\alpha)=1-2\cos\alpha+\cos^2\alpha$,整理得$3\cos^2\alpha-2\cos\alpha=3$.

提升练

一、选择题

1~6.CCDCAC.

提示:

1.因为α为第三象限角,即$2k\pi+\pi<\alpha<2k\pi+\frac{3\pi}{2}$,$k\in\mathbf{Z}$,所以$k\pi+\frac{\pi}{2}<\frac{\alpha}{2}<k\pi+\frac{3\pi}{4}$,$k\in\mathbf{Z}$.当$k$为奇数时,它是第四象限角;当$k$为偶数时,它是第二象限的角.$4k\pi+2\pi<2\alpha<4k\pi+3\pi$,$k\in\mathbf{Z}$,所以$2\alpha$的终边的位置在第一或第二象限或$y$的非正半轴.故选C.

2.选项A,$-210°=-210\times\frac{\pi}{180}=-\frac{7\pi}{6}$,正确;选项B,$405°=405\times\frac{\pi}{180}=\frac{9\pi}{4}$,正确;选项C,$335°=335\times\frac{\pi}{180}=\frac{67\pi}{36}$,错误;选项D,$705°=705\times\frac{\pi}{180}=\frac{47\pi}{12}$,正确.故选C.

3.$\because\alpha$是第三象限角,$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{5}{12}$,$\sin^2\alpha+\cos^2\alpha=1$,得$\sin\alpha=-\frac{5}{13}$.故选D.

4.$\because\cos\alpha-\sin\alpha=\frac{\sqrt{2}}{4}$,$\therefore1-2\sin\alpha\cos\alpha=\frac{1}{8}$,$\therefore2\sin\alpha\cdot\cos\alpha=\frac{7}{8}$,$\therefore\sin\alpha\cos\alpha=\frac{7}{16}$.

故选C.

5.$\because\triangle AOB$的顶角为$120°$,腰长为2,$\therefore A=B=30°=\frac{\pi}{6}$.

方案一中扇形的周长$l_1=2+2+2\times\frac{\pi}{6}=4+\frac{\pi}{3}$,方案二中扇形的周长$l_2=1+1+1\times\frac{2\pi}{3}=2+\frac{2\pi}{3}$;方案一中扇形的面积$S_1=\frac{1}{2}\times2\times2\times\frac{\pi}{6}=\frac{\pi}{3}$,方案二中扇形的面积$S_2=\frac{1}{2}\times1^2\times\frac{2\pi}{3}=\frac{\pi}{3}$,所以$S_1=S_2$,$l_1>l_2$.故选A.

6.$\sqrt{1-2\cos4\sin4}=\sqrt{\cos^2 4+\sin^2 4-2\sin4\cos4}=|\sin4-\cos4|$.$\because\frac{5\pi}{4}<4<\frac{3\pi}{2}$,$\therefore$由三角函数线易知$\cos4>\sin4$.

$\therefore\sqrt{1-2\cos4\sin4}=\cos4-\sin4$.

故选C.

二、填空题

7.$\frac{1}{2}$;

8.$-\frac{\sqrt{5}}{3}$.

提示:

7.由题意知$\frac{1+\sin x}{\cos x}=2$,得$\sin x=2\cos x-1$,代入$\sin^2x+\cos^2x=1$,

解得$\cos x=\frac{4}{5}$,所以$\sin x=\frac{3}{5}$,所以$\frac{1-\sin x}{\cos x}=\frac{1-\frac{3}{5}}{\frac{4}{5}}=\frac{1}{2}$.

故答案为$\frac{1}{2}$.

8.可得$6\sin^2\alpha=5\cos\alpha$,即$6\cos^2\alpha+5\cos\alpha-6=0$,解得$\cos\alpha=\frac{2}{3}$,或$\cos\alpha=-\frac{3}{2}$(舍去).

三、解答题

9.解:(1)由已知,得$\begin{cases}\Delta=k^2-4(k^2-3)=-3k^2+12\geqslant0①,\\ \tan\alpha+\frac{1}{\tan\alpha}=k②,\\ \tan\alpha\cdot\frac{1}{\tan\alpha}=k^2-3③.\end{cases}$

由③得$k^2-3=1$,解得$k=\pm2$,此适合①.

故$k=\pm2$.

(2)注意到α为第三象限角,则$\tan\alpha+\frac{1}{\tan\alpha}>0$.

$\therefore$由(1),得$\tan\alpha+\dfrac{1}{\tan\alpha}=2$,整理得$\tan^2\alpha-2\tan\alpha+1=0$,

解得$\tan\alpha=1$,$\sin\alpha+\cos\alpha=-\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}=-\sqrt{2}$.

10.解:$\because\tan\theta=\sqrt{\dfrac{1-a}{a}}$,

$\therefore\dfrac{\sin^2\theta}{\cos^2\theta}=\dfrac{1-a}{a}=\dfrac{1}{a}-1$,$\therefore a=\cos^2\theta$,

$\therefore\dfrac{\sin^2\theta}{a+\cos\theta}+\dfrac{\sin^2\theta}{a-\cos\theta}=\dfrac{2a\sin^2\theta}{a^2-\cos^2\theta}=\dfrac{2\cos^2\theta\sin^2\theta}{\cos^4\theta-\cos^2\theta}=\dfrac{2\sin^2\theta\cos^2\theta}{\cos^2\theta(\cos^2\theta-1)}=\dfrac{2\sin^2\theta\cos^2\theta}{-\sin^2\theta\cos^2\theta}=-2$.

38期《堂堂清》

1.3 诱导公式二

1.$\sqrt{3}$; 2.$-\dfrac{1}{2}$; 3.D; 4.$-\dfrac{4}{5}$; 5.C; 6.C.

1.3 诱导公式三

1.C; 2.$-\dfrac{\sqrt{3}}{2}$; 3.-1; 4.B; 5.-1.

1.3 诱导公式四

1.$-\dfrac{\sqrt{3}}{3}$; 2.$\dfrac{1}{2}$; 3.A; 4.D; 5.B; 6.C;

7.$-\dfrac{3}{2}$; 8.D.

1.3 诱导公式五

1.a; 2.$\dfrac{2}{3}$; 3.$\sqrt{1-b^2}$; 4.D;

5.$\sin\alpha-\cos\alpha$.

6.解:注意到$(\dfrac{\pi}{6}-\alpha)+(\dfrac{\pi}{3}+\alpha)=\dfrac{\pi}{2}$,所以$\cos(\dfrac{\pi}{3}+\alpha)=\cos[\dfrac{\pi}{2}-(\dfrac{\pi}{6}-\alpha)]=\sin(\dfrac{\pi}{6}-\alpha)=\dfrac{1}{3}$.

1.3 诱导公式六

1.B; 2.B; 3.$-m$; 4.$\dfrac{1}{5}$; 5.D; 6.$\dfrac{3}{5}$.

同步分层能力测试题(二)

基础练

一、选择题

1~6.BABBDC.

提示:

1.$\cos570°=\cos(360°+210°)=\cos210°=\cos(180°+30°)=-\cos30°=-\dfrac{\sqrt{3}}{2}$.

故选B.

2.$\because-\cos\alpha=\cos(\pi-\alpha)=-\dfrac{5}{13}$,$\therefore\cos\alpha=\dfrac{5}{13}$.又$\because\alpha$是第四象限,$\therefore\sin\alpha=-\sqrt{1-\cos^2\alpha}=-\dfrac{12}{13}$.

故选A.

3.$\sin(\dfrac{3\pi}{2}-\alpha)=-\cos\alpha$,而$r=|OA|=\sqrt{5}$,所以$\cos\alpha=\dfrac{2\sqrt{5}}{5}$,故$\sin(\dfrac{3\pi}{2}-\alpha)=-\dfrac{2\sqrt{5}}{5}$.故选B.

4.由题$\cos(\pi+\alpha)=-\dfrac{1}{3}$,且$\alpha$为第四象限角,则$\cos(\pi+\alpha)=-\cos\alpha=-\dfrac{1}{3}$,$\therefore\cos\alpha=\dfrac{1}{3}$,$\sin\alpha=-\dfrac{2\sqrt{2}}{3}$,$\sin(2\pi-\alpha)=-\sin\alpha=\dfrac{2\sqrt{2}}{3}$.

故选B.

5.$\because A+B+C=\pi$,$\therefore A+B=\pi-C$,$\therefore\sin(A+B)-\sin C=\sin(\pi-C)-\sin C=\sin C-\sin C=0$,为常数;

$\cos(A+B)+\cos C=\cos(\pi-C)+\cos C=-\cos C+\cos C=0$,为常数;

$\tan(A+B)+\tan C=\tan(\pi-C)+\tan C=-\tan C+\tan C=0$,为常数.

故选D.

6.根据三角函数的诱导公式和三角函数基本关系式,可得$\dfrac{\sin(\pi+\alpha)}{2\sin\alpha+3\cos(-\alpha)}=\dfrac{-\sin\alpha}{3\sin\alpha+3\cos\alpha}=\dfrac{-\tan\alpha}{2\tan\alpha+3}=-\dfrac{2}{5}$,解得$\tan\alpha=6$.故选C.

二、填空题

7.$\dfrac{12}{13}$;

8.$-2\sqrt{2}$.

提示:

7.$\because\cos(\alpha-\dfrac{\pi}{6})=\cos(\alpha+\dfrac{\pi}{3}-\dfrac{\pi}{2})=\sin(\alpha+\dfrac{\pi}{3})=\dfrac{12}{13}$.故答案为$\dfrac{12}{13}$.

8.$\because-\dfrac{\pi}{2}<\alpha<0$,$\therefore\sin\alpha=-\sqrt{1-\dfrac{1}{9}}=-\dfrac{2\sqrt{2}}{3}$,

$\therefore$原式$=\dfrac{\cos\alpha\sin\alpha\tan\alpha}{\cos\alpha\sin\alpha}=\dfrac{\sin\alpha}{\cos\alpha}=-2\sqrt{2}$.

三、解答题

9.解:$\because\sin\alpha+\cos\alpha=\dfrac{\sqrt{2}}{3}$①,将①两边平方,得$1+2\sin\alpha\cdot\cos\alpha=\dfrac{2}{9}$,故$2\sin\alpha\cdot\cos\alpha=-\dfrac{7}{9}$.又$\dfrac{\pi}{2}<\alpha<\pi$,$\therefore\sin\alpha>0$,$\cos\alpha<0$.

(1)$(\sin\alpha-\cos\alpha)^2=1-2\sin\alpha\cdot\cos\alpha=1-(-\dfrac{7}{9})=\dfrac{16}{9}$,

$\therefore\sin\alpha-\cos\alpha=\dfrac{4}{3}$.

(2)$\sin^2(\dfrac{\pi}{2}-\alpha)-\cos^2(\dfrac{\pi}{2}+\alpha)=\cos^2\alpha-\sin^2\alpha=(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)=-\dfrac{4}{3}\times\dfrac{\sqrt{2}}{3}=-\dfrac{4\sqrt{2}}{9}$.

10.解:由已知,得$\cos x=-\dfrac{2}{3}$,故$\sin(2y+x)=\sin[2(k\pi+\dfrac{\pi}{4}-x)+x]=\sin(\dfrac{\pi}{2}-x)=\cos x=-\dfrac{2}{3}$.

11.解:(1)$\sin(\pi+\alpha)=-\sin\alpha=-\dfrac{15m}{17|m|}$,所以当$m>0$时,原式$=-\dfrac{15}{17}$;当$m<0$时,原式$=\dfrac{15}{17}$.

(2)$\dfrac{\sin(\pi+\alpha)\cos(-\alpha-\dfrac{\pi}{2})\tan(\alpha-5\pi)}{\cos(\alpha-\dfrac{3\pi}{2})}$

$=\dfrac{-\sin\alpha\cdot(-\sin\alpha)\cdot\tan\alpha}{-\sin\alpha}=-\dfrac{15m}{17|m|}\cdot\dfrac{15}{8}$,

所以当$m>0$时,原式$=-\dfrac{225}{136}$;当$m<0$时,原式$=\dfrac{225}{136}$.

12.解:$f(\alpha)=\dfrac{\sin(\pi-\alpha)\cos(2\pi-\alpha)\cos(\dfrac{3\pi}{2}+\alpha)}{\cos(\dfrac{\pi}{2}+\alpha)\sin(\pi+\alpha)}$

$=\dfrac{\sin\alpha\cos\alpha\sin\alpha}{(-\sin\alpha)(-\sin\alpha)}=\cos\alpha$.

(1)$f(-\dfrac{13\pi}{3})=\cos(-\dfrac{13\pi}{3})=\cos\dfrac{\pi}{3}=\dfrac{1}{2}$.

(2)$\because\cos(\dfrac{\pi}{2}-\alpha)=\dfrac{3}{5}$,$\therefore\sin\alpha=\dfrac{3}{5}$.$\because\alpha$是第二象限角,$\therefore\cos\alpha=-\dfrac{4}{5}$,$\therefore f(\alpha)=\cos\alpha=-\dfrac{4}{5}$.

提升练

一、选择题

1~6.DBBADC.

提示:

1.根据诱导公式,$\cos(\pi+A)=-\cos A=-\dfrac{1}{2}$,所以$\cos A=\dfrac{1}{2}$.而$\sin(\dfrac{\pi}{2}+A)=\cos A=\dfrac{1}{2}$,故选D.

2.$\cos^21°+\cos^22°+\cdots+\cos^288°+\cos^289°=\cos^21°+\cos^22°+\cdots+\cos^244°+\cos^245°+\sin^244°+\cdots+\sin^22°+\sin^21°=(\cos^21°+\sin^21°)+(\cos^22°+\sin^22°)+\cdots+(\cos^244°+\sin^244°)+\cos^245°=44+0.5=44.5$.

故选B.

3.$\because\tan(\alpha-7\pi)=\tan\alpha=-\dfrac{3}{4}$,$\therefore\alpha\in(\dfrac{\pi}{2},\pi)$,$\therefore\sin\alpha=\dfrac{3}{5}$,$\cos\alpha=-\dfrac{4}{5}$,$\therefore\sin\alpha+\cos\alpha=-\dfrac{1}{5}$.

故选B.

4.注意到$\dfrac{\pi}{2}<2<\pi$,则原式$=|\sin2-\cos2|=\sin2-\cos2$.

故选A.

5.已知θ为第一象限角,角θ的终边与单位圆的交点坐标为$(\cos\theta,\sin\theta)$,将角θ的终边逆时针旋转$\dfrac{\pi}{2}$,得到角$\theta+\dfrac{\pi}{2}$,角$\theta+\dfrac{\pi}{2}$的终边与单位圆的交点坐标为$(\cos(\theta+\dfrac{\pi}{2}),\sin(\theta+\dfrac{\pi}{2}))$,即$(-\sin\theta,\cos\theta)$.故选D.

6.$f(2010\pi-3)=-a\tan\dfrac{3}{2}+b\sin3+4=-f(3)+8=-5+8=3$.

故选C.

二、填空题

7.0;

8.$\dfrac{2}{5}$.

提示:

7.$\cos(\dfrac{5\pi}{6}+\theta)+\sin(\dfrac{2\pi}{3}-\theta)=\cos[\pi-(\dfrac{\pi}{6}-\theta)]+\sin[\dfrac{\pi}{2}+(\dfrac{\pi}{6}-\theta)]=-\cos(\dfrac{\pi}{6}-\theta)+\cos(\dfrac{\pi}{6}-\theta)=-a+a=0$.故答案为:0.

8.$\because\tan\alpha=2$,则$\sin\alpha\sin(\dfrac{\pi}{2}-\alpha)=\sin\alpha\cdot\cos\alpha=\dfrac{\sin\alpha\cos\alpha}{\sin^2\alpha+\cos^2\alpha}=\dfrac{\tan\alpha}{\tan^2\alpha+1}=\dfrac{2}{5}$.

三、解答题

9.解:(1)证明:由已知,得$\sin A=\sqrt{2}\sin B$①,$\sqrt{3}\cos A=\sqrt{2}\cos B$②.

由$①^2+②^2$,有$\sin^2A+3\cos^2A=2$,即$1-\cos^2A+3\cos^2A=2$,得$\cos A=\pm\dfrac{\sqrt{2}}{2}$.

若$\cos A=-\dfrac{\sqrt{2}}{2}$,则由②,得$\cos B=-\dfrac{\sqrt{3}}{2}$,此时$A$,$B$均为钝角,不可能,$\therefore\cos A=\dfrac{\sqrt{2}}{2}$.

命题得证.

(2)将$\cos A=\dfrac{\sqrt{2}}{2}$代入②,得$\cos B=\dfrac{\sqrt{3}}{2}$.

注意到$A,B\in(0,\pi)$,则$A=\dfrac{\pi}{4}$,$B=\dfrac{\pi}{6}$,$C=\pi-(A+B)=\dfrac{7\pi}{12}$为所求.

10.解:(1)$f(\alpha)=\dfrac{\cos\alpha(2\sin\alpha+1)}{1+\sin^2\alpha+\sin\alpha-\cos^2\alpha}=\dfrac{\cos\alpha(2\sin\alpha+1)}{2\sin^2\alpha+\sin\alpha}=\dfrac{\cos\alpha(2\sin\alpha+1)}{\sin\alpha(2\sin\alpha+1)}=\dfrac{\cos\alpha}{\sin\alpha}$.

(2)原式$=\dfrac{\cos1°}{\sin1°}\cdot\dfrac{\cos2°}{\sin2°}\cdot\cdots\cdot\dfrac{\cos45°}{\sin45°}\cdot\cdots\cdot\dfrac{\cos88°}{\sin88°}\cdot\dfrac{\cos89°}{\sin89°}=(\dfrac{\cos1°}{\sin1°}\cdot\dfrac{\cos89°}{\sin89°})\cdot(\dfrac{\cos2°}{\sin2°}\cdot\dfrac{\cos88°}{\sin88°})\cdot\cdots\cdot\dfrac{\cos45°}{\sin45°}=(\dfrac{\cos1°}{\sin1°}\cdot\dfrac{\sin1°}{\cos1°})\cdot(\dfrac{\cos2°}{\sin2°}\cdot\dfrac{\sin2°}{\cos2°})\cdot\cdots\cdot\dfrac{\cos45°}{\sin45°}=1$.

39期《堂堂清》

1.4.1 正弦函数、余弦函数的图象

1.D; 2.$(0,\pi)$; 3.A; 4.B; 5.C;

6.$(0,0)$,$(\dfrac{\pi}{2},2)$,$(\pi,0)$,$(\dfrac{3\pi}{2},-2)$,$(2\pi,0)$.

7.解:$y=\sin x-|\sin x|=\begin{cases}0, & \sin x>0,\\ 2\sin x, & \sin x<0,\end{cases}$由此得其值域为$[-2,0]$.

1.4.2 正弦函数、余弦函数的性质

1.B; 2.B; 3.2π; 4.A; 5.B. 6.A;

7.B; 8.A; 9.C; 10.A;

11.$[k\pi+\dfrac{\pi}{4},k\pi+\dfrac{3\pi}{4}]$,$k\in\mathbf{Z}$.

1.4.3 正切函数的性质与图象

1.C; 2.<; 3.A; 4.B; 5.A. 6.C; 7.A; 8.$\dfrac{\pi}{\omega}$.

同步分层能力测试题(三)

基础练

一、选择题

1~6.ACBCCB.

提示:

1.$\alpha\in(2k\pi+\dfrac{\pi}{4},2k\pi+\dfrac{\pi}{2})(k\in\mathbf{Z})\Rightarrow1>\sin\alpha>\cos\alpha$,$\tan\alpha>1$,故$\tan\alpha>\sin\alpha>\cos\alpha$.故选A.

2.对A选项,令$x=\dfrac{3\pi}{2}$,则$f(-\dfrac{3\pi}{2})=\sin|-\dfrac{3\pi}{2}|=-1$,$f(-\dfrac{3\pi}{2}+\pi)=\sin|-\dfrac{\pi}{2}|=1$不满足$f(-\dfrac{3\pi}{2})=f(-\dfrac{3\pi}{2}+\pi)$,所以$y=\sin|x|$不是以$\pi$为周期的函数,其最小正周期不为$\pi$;对B选项,$y=1+\sin x$的最小正周期为$T=2\pi$;对D选项,$y=\tan2x$的最小正周期为$T=\dfrac{\pi}{2}$;排除A,B,D.故选C.

3.由$x-\dfrac{\pi}{4}=\dfrac{\pi}{2}+k\pi(k\in\mathbf{Z})$,得出当$k=1$时,$x=\dfrac{7}{4}\pi$.

故选B.

4.由$2x+\dfrac{\pi}{4}\neq k\pi+\dfrac{\pi}{2}$,$k\in\mathbf{Z}$,得$x\neq\dfrac{\pi}{8}+\dfrac{k\pi}{2}$,$k\in\mathbf{Z}$.$\therefore f(x)$的定义域为$\{x|x\neq\dfrac{\pi}{8}+\dfrac{k\pi}{2},k\in\mathbf{Z}\}$.

故选C.

5.依题意,得$\Delta=16\sin^2\theta-24\cos\theta<0$,解得$\cos\theta>\dfrac{1}{2}$,或$\cos\theta<-2$(舍去).

又$0<\theta<\pi$,故有$\theta\in(0,\dfrac{\pi}{3})$.

故选C.

6.A:$f(x)=\tan x$的最小正周期为π,所以2π是$f(x)$的一个周期,所以该选项正确;B:$f(-\dfrac{3\pi}{4})=1$,$f(\dfrac{3\pi}{4})=-1$,所以该选项是错误的;C:$f(x)=\tan x$的值域为$\mathbf{R}$,所以该选项是正确的;D:$f(x)=\tan x$的图象关于点$(\dfrac{\pi}{2},0)$对称,所以该选项是正确的.故选B.

二、填空题

7.π;$[-\dfrac{\pi}{8}+k\pi,\dfrac{3\pi}{8}+k\pi](k\in\mathbf{Z})$;

8.2.

提示:

7.因为$f(x)=\sin(2x-\dfrac{\pi}{4})$,所以$T=\dfrac{2\pi}{|\omega|}=\dfrac{2\pi}{2}=\pi$.因为$-\dfrac{\pi}{2}+2k\pi\leqslant2x-\dfrac{\pi}{4}\leqslant\dfrac{\pi}{2}+2k\pi\Rightarrow-\dfrac{\pi}{8}+k\pi\leqslant x\leqslant\dfrac{3\pi}{8}+k\pi$$(k\in\mathbf{Z})$,所以增区间为$[-\dfrac{\pi}{8}+k\pi,\dfrac{3\pi}{8}+k\pi](k\in\mathbf{Z})$.

8.$x\in[0,\dfrac{\pi}{2}]$时,$2x+\dfrac{\pi}{3}\in[\dfrac{\pi}{3},\dfrac{4\pi}{3}]$,则$\cos(2x+\dfrac{\pi}{3})\in[-1,\dfrac{1}{2}]$.由于$a>0$,所以函数$y=a\cos(2x+\dfrac{\pi}{3})+3$的最大值为$\dfrac{1}{2}a+3=4$,则$a=2$.

三、解答题

9.解:(1)$f(x)$的周期为$T=\dfrac{2\pi}{2}=\pi$.

(2)由$x\in[\dfrac{\pi}{8},\dfrac{3\pi}{4}]$,有$2x-\dfrac{\pi}{4}\in[0,\dfrac{5\pi}{4}]$,

得$\sin(2x-\dfrac{\pi}{4})\in[-\dfrac{\sqrt{2}}{2},1]$.

故$f(x)$的值域为$[-1,\sqrt{2}]$.

10.解:(1)因为函数$f(x)=1+\dfrac{1}{2}\sin2x$,$\therefore T=\dfrac{2\pi}{2}=\pi$.

令$\dfrac{\pi}{2}+2k\pi\leqslant2x\leqslant\dfrac{3\pi}{2}+2k\pi(k\in\mathbf{Z})$,

则$\dfrac{\pi}{4}+k\pi\leqslant x\leqslant\dfrac{3\pi}{4}+k\pi(k\in\mathbf{Z})$,

即函数$f(x)$的单调递减区间是$[\dfrac{\pi}{4}+k\pi,\dfrac{3\pi}{4}+k\pi]$$(k\in\mathbf{Z})$.

(2)因为$\sin2x=2\sin x\cos x$,

所以$f(x)=\dfrac{\sin^2x+\sin x\cos x+\cos^2x}{\sin^2x+\cos^2x}=\dfrac{\tan^2x+\tan x+1}{\tan^2x+1}$,

$\therefore$当$\tan x=2$时,$f(x)=\dfrac{2^2+2+1}{2^2+1}=\dfrac{7}{5}$.

11.解:(1)因为函数$f(x)=\sqrt{2}\sin(2x-\dfrac{\pi}{4})-1+m$的图象经过点$(\dfrac{\pi}{8},0)$,

$\therefore f(\dfrac{\pi}{8})=\sqrt{2}\sin(2\times\dfrac{\pi}{8}-\dfrac{\pi}{4})-1+m=m-1=0$,

所以$m=1$,

$\therefore f(x)=\sqrt{2}\sin(2x-\dfrac{\pi}{4})$,

所以当$2x-\dfrac{\pi}{4}=\dfrac{\pi}{2}+2k\pi$,$k\in\mathbf{Z}$,即$x=\dfrac{3\pi}{8}+k\pi$,$k\in\mathbf{Z}$时,$f(x)$取到最大值$\sqrt{2}$.

(2)$f(\dfrac{\alpha}{2})=\sqrt{2}\sin(\alpha-\dfrac{\pi}{4})=\dfrac{3\sqrt{2}}{5}$,

$\therefore\sin(\alpha-\dfrac{\pi}{4})=\dfrac{3}{5}$.

$\because\alpha\in(0,\dfrac{\pi}{2})$,$\therefore\alpha-\dfrac{\pi}{4}\in(-\dfrac{\pi}{4},\dfrac{\pi}{4})$,

$\therefore\cos(\alpha-\dfrac{\pi}{4})=\sqrt{1-\sin^2(\alpha-\dfrac{\pi}{4})}=\dfrac{4}{5}$,

$\therefore\tan(\alpha-\dfrac{\pi}{4})=\dfrac{\sin(\alpha-\dfrac{\pi}{4})}{\cos(\alpha-\dfrac{\pi}{4})}=\dfrac{\dfrac{3}{5}}{\dfrac{4}{5}}=\dfrac{3}{4}$.

12.解:本题答案不唯一.例如:(1)偶函数;(2)最大值为3;(3)在$[0,\dfrac{\pi}{4}]$上是增函数等.

提升练

一、选择题

1~6.ADBDAA.

提示:

1.直线$y=c(c\in\mathbf{R})$与函数$y=\tan\omega x(\omega\neq0)$的图象相邻的两个交点之间的距离为1,则$T=1\Rightarrow\omega=\pi\Rightarrow y=\tan\pi x$,$y=\tan\pi x$的对称中心横坐标为$\pi x=\dfrac{k\pi}{2}\Rightarrow x=\dfrac{k}{2}(k\in\mathbf{Z})$,即对称中心为$(\dfrac{k}{2},0)$,$k\in\mathbf{Z}$.故选A.

2.由题意可知$\theta+\dfrac{\pi}{6}=2k\pi+\dfrac{\pi}{2}$,$k\in\mathbf{Z}$,所以$\theta=2k\pi+\dfrac{\pi}{3}$,$k\in\mathbf{Z}$,$\therefore\tan\theta=\sqrt{3}$.

故选D.

3.当$x=\dfrac{\pi}{8}$时,$y=0$.

故选B.

4.因为对任意的$x\in\mathbf{R}$,$f(x_1)\leqslant f(x)\leqslant f(x_2)$成立,

所以$f(x_1)=f(x)_{\min}=-3$,$f(x_2)=f(x)_{\max}=3$,所以$|x_1-x_2|_{\min}=\dfrac{T}{2}=2$.故选D.

5.画出$y=\sin x$,$y=m$在$[0,\pi]$上的图象,知两交点关于直线$x=\dfrac{\pi}{2}$对称,则$\dfrac{x_1+x_2}{2}=\dfrac{\pi}{2}$,得$x_1+x_2=\pi$.

故选A.

6.$\because\sin x+\sin y=\dfrac{1}{3}$,$\therefore\sin y=\dfrac{1}{3}-\sin x\in[-1,1]$,$\therefore\sin x\in[-\dfrac{2}{3},1]$,则$u=\sin x+1-\sin^2x=-(\sin x-\dfrac{1}{2})^2+\dfrac{5}{4}$.结合二次函数的图象可知,当$\sin x=-\dfrac{2}{3}$时,函数取得最小值且为$-\dfrac{1}{9}$.

故选A.

二、填空题

7.$[-\dfrac{\sqrt{3}}{3},\dfrac{\sqrt{3}}{3}]$;

8.$[k\pi+\dfrac{3\pi}{8},k\pi+\dfrac{7\pi}{8}](k\in\mathbf{Z})$.

提示:

7.由$y=\dfrac{\sin x}{\cos x-2}$,得定义域为$\mathbf{R}$,且$\sin x=y\cos x-2y$,即有$\sin(x+\varphi)=\dfrac{-2y}{\sqrt{1+y^2}}$,所以$|\sin(x+\varphi)|=|\dfrac{-2y}{\sqrt{1+y^2}}|\leqslant1$,解得$y\in[-\dfrac{\sqrt{3}}{3},\dfrac{\sqrt{3}}{3}]$,故函数的值域为$[-\dfrac{\sqrt{3}}{3},\dfrac{\sqrt{3}}{3}]$.

8.函数$y=\sin(\dfrac{\pi}{4}-2x)=-\sin(2x-\dfrac{\pi}{4})$,由$\dfrac{\pi}{2}+2k\pi\leqslant2x-\dfrac{\pi}{4}\leqslant\dfrac{3\pi}{2}+2k\pi$,$k\in\mathbf{Z}$,解得$\dfrac{3\pi}{8}+k\pi\leqslant x\leqslant\dfrac{7\pi}{8}+k\pi$,$k\in\mathbf{Z}$,所以函数$y=\sin(\dfrac{\pi}{4}-2x)$的增区间是$[k\pi+\dfrac{3\pi}{8},k\pi+\dfrac{7\pi}{8}]$$(k\in\mathbf{Z})$.故答案为$[k\pi+\dfrac{3\pi}{8},k\pi+\dfrac{7\pi}{8}](k\in\mathbf{Z})$.

三、解答题

9.解:(1)令$t=\sin(2x-\dfrac{\pi}{3})$,由$t>0$,得$2k\pi<2x-\dfrac{\pi}{3}<2k\pi+\pi$,$k\in\mathbf{Z}$,得$k\pi+\dfrac{\pi}{6}<x<k\pi+\dfrac{2\pi}{3}$,$k\in\mathbf{Z}$.

故$f(x)$的定义域为$(k\pi+\dfrac{\pi}{6},k\pi+\dfrac{2\pi}{3})(k\in\mathbf{Z})$.

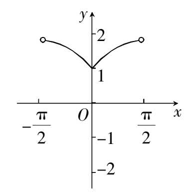
(2)注意到$y=\ln t$递增,$f(x)$递减,则$t=\sin(2x-\dfrac{\pi}{3})$递减.

结合(1),有$2k\pi+\dfrac{\pi}{2}\leqslant2x-\dfrac{\pi}{3}<2k\pi+\pi$,$k\in\mathbf{Z}$,

得$k\pi+\dfrac{5\pi}{12}\leqslant x<k\pi+\dfrac{2\pi}{3}$,$k\in\mathbf{Z}$.

故$f(x)$的递减区间为$[k\pi+\dfrac{5\pi}{12},k\pi+\dfrac{2\pi}{3})(k\in\mathbf{Z})$.

10.解:(1)当$-\dfrac{\pi}{2}<x<0$时,$f(x)=\dfrac{1-\sin^2x}{1+\sin x}=1-\sin x$;当$0\leqslant x<\dfrac{\pi}{2}$时,$f(x)=1+\sin x$,即$f(x)=\begin{cases}1-\sin x, & -\dfrac{\pi}{2}<x<0,\\ 1+\sin x, & 0\leqslant x<\dfrac{\pi}{2},\end{cases}$故函数$f(x)$的草图如下:



(2)由(1),可得$f(x)$的值域为$[1,2)$,$f(x)$为偶函数,且$f(x)$在$(-\dfrac{\pi}{2},0)$上是减函数,在$[0,\dfrac{\pi}{2})$上是增函数.

40期《堂堂清》

1.5 函数$y=A\sin(\omega x+\varphi)$的图象

1.C;

2.纵坐标伸长到原来的3倍(横坐标不变);

3.C; 4.C; 5.C; 6.C; 7.$\dfrac{\pi}{4}$; 8.D; 9.C; 10.B;

11.$y=\sin(\dfrac{1}{2}x+\dfrac{\pi}{6})$,$x\in\mathbf{R}$.

12.$f(x)=2\sin(2x+\dfrac{\pi}{6})$.

1.6 三角函数模型的简单应用

1.$\dfrac{1}{50}$; 2.3; 3.B; 4.B; 5.20°C; 6.3;6;1.

7.5安; 8.B.